Physics 201 Name KEY

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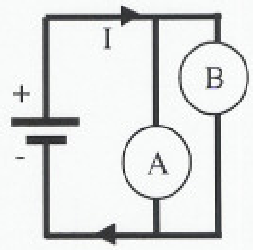
Exam 2 – Circuits

March 1, 2007

This is a closed book examination. However, you may use a 4x5 index you have created on this exam. There is extra scratch paper available. Please explain your answers. Your explanation is worth 3/4 of the points on multiple-choice questions.

1) [4 PTS] Two light bulbs are connected to a battery as shown in the diagram to the right. Bulb A is brighter than bulb B. What happens to the brightness when you switch the location of bulb A and bulb B?

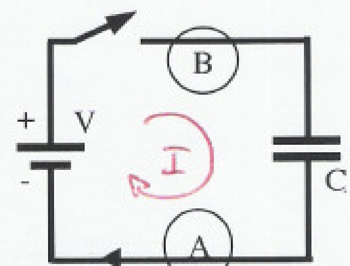


a) Bulb A is still brighter
b) Bulb B is now brighter
c) Depends on direction of current
d) Bulb B and Bulb A are now the same brightness

Current is still the same since voltage is the same $V = IR$

$P = IV = \frac{V^2}{R}$ so no change

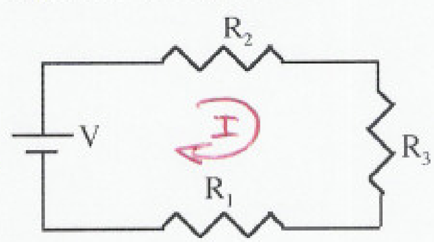
2) [4 PTS] In the circuit to the right the capacitor is initially uncharged. If you want a light bulb to light when you throw the switch, where should you place it?



a) Insert the light bulb at A.
b) Insert the light bulb at B.
c) Either position A or B will light the bulb.
d) Bulb will not light in this circuit. Neither position A or B will light the bulb.

Current will flow around entire circuit. Current is not restricted to flow on just the top or bottom

3) [4 PTS] You connect three resistors to a battery as shown in the diagram to the right. The current through each resistor



a) is the same.
b) is different.
c) is zero.
d) depends on the polarity of the battery.
e) depends on which resistor is the smallest.

In a series circuit the current has no choice – it flows through each circuit element

4) [4 PTS] A wire with a resistance, R_1, is replaced by a new wire that is made out of the same material and is the same length but has twice the cross-sectional radius. What is the resistance of this new wire, R_2?

a) Smaller, $R_2 = \frac{1}{4}R_1$.
b) Smaller, $R_1 = \frac{1}{2}R_1$.
c) The same, $R_2 = R_1$.
d) Larger, $R_2 = 2R_1$.
e) Larger, $R_2 = 4R_1$.

$R = \rho\frac{L}{A} = \frac{\rho L}{\pi r^2}$ so $R(r) \propto \frac{1}{r^2}$

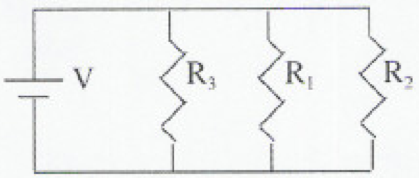
Resistance decreases – $r_2 = 2r_1$ so

$R_2 = \frac{1}{4}R_1$

$\left(R_2 = \rho\frac{L}{\pi r_2^2} = \frac{\rho L}{\pi(2r_1)^2} = \frac{\rho L}{4\pi r_1^2} = \frac{1}{4}R_1\right)$

KEY

5) [4 PTS] You connect three resistors to a battery as shown in the diagram to the right. The current through each resistor



a) is the same.
b) is different.
c) is zero.
d) depends on the polarity of the battery.
e) depends on which resistor is the smallest.

$V = IR$

Voltage drop across each resistor is the same...

$I = \frac{V}{R}$ so $I(R)$

6) [4 PTS] For the above circuit (problem #5) R_1=50Ω, R_2=500Ω and R_3=350Ω. The voltage across R_1

a) is greater than the voltage across R_2.
b) is less than the voltage across R_3.
c) is less than V.
d) depends on the current through R_2 and R_3.
e) is equal to V.

Voltage drop across all resistors is the same.

$V_1 = V_2 = V_3 = V$

7) [4 PTS] For the above circuit (problem #5) what is the total resistance for the circuit?

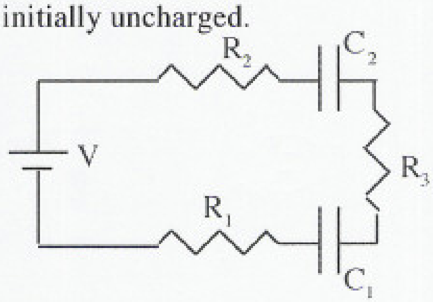
a) $R_3 > R_{total} > R_1$
b) $R_2 > R_{total} > R_3$
c) $R_{total} > R_2$
d) $R_1 > R_{total}$
e) None of the above

$\frac{1}{R_{TOT}} = \sum_i \frac{1}{R_i}$

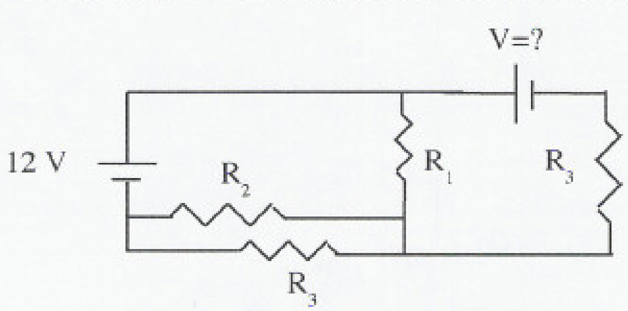
R_{TOT} is less than the smallest resistor — you are adding more paths so resistance has to decrease.

8) [12 PTS] You construct the following circuit with R_1=50Ω, R_2=500Ω, R_3=350Ω, C_1=400μF, C_2=500μF and V=9Volts. The capacitors are initially uncharged.

a) What is the time constant?
b) Which resistor has the largest voltage drop across it?
c) Graph the voltage across C_2 as a function of time.
d) Graph the voltage across R_3 as a function of time.



9) [8 PTS] Solve for the unknown source voltage and the power delivered by the unknown battery in the diagram below. The current through R_1 is 1 Ampere while R_1 = 9 Ω, R_2 = 3 Ω and R_3 = 1 Ω. NOTE: A correct setup is worth 6 PTS.



[8]

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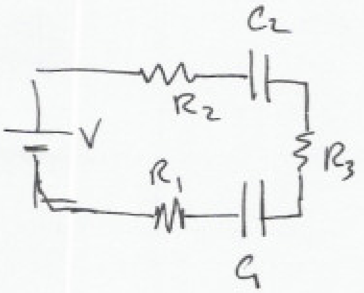


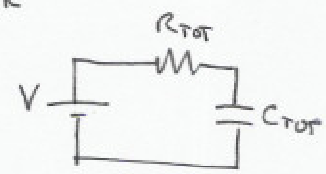
$V = 9V$

$R_1 = 50\Omega \quad R_2 = 500\Omega \quad R_3 = 350\Omega \quad R$

$R_{TOT} = \sum R_i = 900\Omega$

$C_1 = 400\mu F \quad C_2 = 500\mu F$

$C_{TOT} = \left(\sum \frac{1}{C_i}\right)^{-1} = \frac{1}{4500} F \quad (222\mu F)$



[Simplify the circuit]

(a) $\tau = RC = 900\Omega \cdot \frac{1}{4500} F = \frac{1}{5} s \quad (0.2\,s)$

(b) Current is ~~constant~~ same through all the circuit elements $V = IR$ so the resistor with the largest value needs the most voltage

$R_2 = 500\Omega$ has largest voltage drop

(c) $Q = C_{TOT} V_c$ and $I R_{TOT} = V_R$

$\frac{I(t)}{C} = \frac{d}{dt}(V_c(t))$ for each capacitor

to find $V_c(t)$ integrate

$V_c(t) = \int \frac{I(t)}{C_2} dt$

$= \frac{1}{C_2} \int I_0 e^{-t/R_{TOT}C_{TOT}} dt$

$= \frac{I_0}{C_2} \int e^{-t/R_{TOT}C_{TOT}} dt$

$= \frac{I_0}{C_2} \cdot -R_{TOT}C_{TOT}\, e^{-t/R_{TOT}C_{TOT}} + V_{const}$

— Derivation —

$V = V_R + V_c$

$V = IR_{TOT} + \frac{Q}{C_{TOT}}$

$\frac{dV}{dt} = 0 = \frac{d}{dt}\left(IR_{TOT} + \frac{Q}{C_{TOT}}\right) = R_{TOT}\frac{dI}{dt} + \frac{1}{C_{TOT}}\frac{dQ}{dt}$

$0 = R_{TOT}C_{TOT}\frac{dI}{dt} + I$

solution is exponential function

$I(t) = I_0 e^{-t/RC}$

Initially $V_c = 0$ so $I_0 = \frac{V_B}{R_{TOT}} = \frac{9V}{900\Omega}$

$I(t) = 0.01\, e^{-t/0.2s}$ Amps

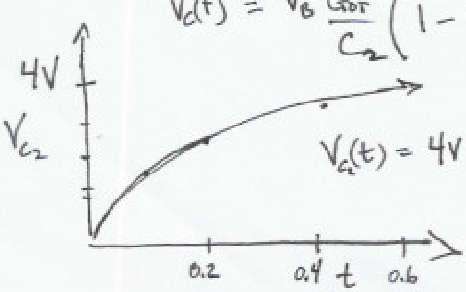
$V(0) = 0$ so $V_{const} = \frac{I_0 R_{TOT} C_{TOT}}{C_2}$

$V_c(t) = I_0 R_{TOT} \frac{C_{TOT}}{C_2}\left(1 - e^{-t/R_{TOT}C_{TOT}}\right)$

or

$V_c(t) = V_B \frac{C_{TOT}}{C_2}\left(1 - e^{-t/\tau}\right)$

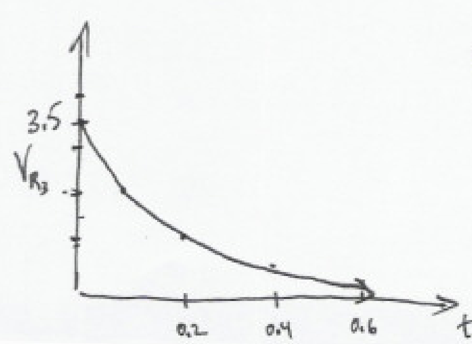
Check units
Check ΣV
Check Behavior $t\uparrow$ $V_R\downarrow$ $I\downarrow$



$V_{C_2}(t) = 4V\left(1 - e^{-t/0.2s}\right)$

(d) ~~$V_3 = IR_3$~~

$V_{R_3}(t) = \frac{V_B}{R_{TOT}} R_3\, e^{-t/\tau} = 3.5V e^{-t/0.2s}$

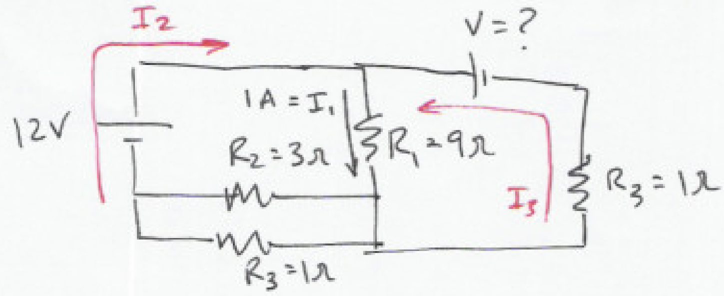


NOTE!

$V_{R_2}(t) = 5V e^{-t/0.2s}$

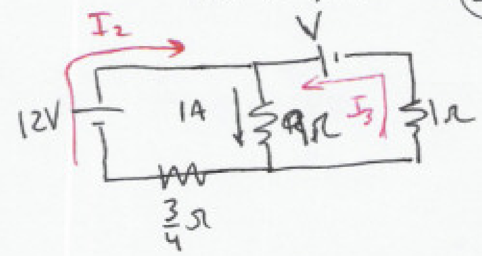
$V_{R_1}(t) = 0.5V e^{-t/0.2s}$

$V_{C_1}(t) = 5V\left(1 - e^{-t/0.2s}\right)$



Simplify the circuit – namely R_2 is parallel to R_3

$$3\Omega // 1\Omega = \left(\sum \frac{1}{3\Omega} + \frac{1}{1\Omega}\right)^{-1} = \frac{3}{4}\Omega$$



(1) $1A = I_2 + I_3$

(2) $12V - (1A)(9\Omega) - I_2\left(\frac{3}{4}\Omega\right) = 0$ $\Rightarrow$ $I_2 = \dfrac{3V}{\frac{3}{4}\Omega} = \boxed{4A = I_2}$

(3) $V - (1A)(9\Omega) - I_3(1\Omega) = 0$

$\Rightarrow$ Then from equation (1)

$1A = 4A + I_3$ $\boxed{I_3 = -3A}$

I picked the wrong direction initially so this means current is being forced through the unknown battery

$\Rightarrow$ from equation (3)

$V = 9V + (-3A)(1\Omega)$

$\boxed{V = 6V}$

$P = VI$

$P = (6V)(-3A)$

$= -18W$

The battery is being charged!!

☐ Check units
☐ Check $\sum V = 0$
☐ Check $\sum I = 0$

(Do these for your new values (w/ current current directions)